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Influence of particle shape on granular contact signatures and shear strength: new insights from simulations

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Abstract

The transmission of force within granular materials is sensitive to local particle arrangements, and because of this strong dependence, contact forces will usually be distributed in a complex, non-uniform manner. Load is transmitted by relatively rigid, heavily stressed chains of particles which form a sparse network of "strong" contacts carrying greater than average normal contact forces. The remaining groups of particles, which separate the strong force chains, are only lightly loaded. Based on computer simulations, we show that this complexity in three-dimensional granular materials can be greatly resolved by considering a single, gross measure of the anisotropic fabric distribution of strong contacts.

Keywords: Granular media; Contact; Stress; Heterogeneous materials

1. Introduction

The transmission of force within unbonded particulate materials occurs only through the contacts among particles, but each contact force is highly sensitive to the local arrangement of surrounding particles (Drescher, 1972; Oda and Konishi, 1974; Liu et al., 1995; Howell et al., 1999; Mueth et al., 2000). Because of this strong dependence on particle arrangement, contact forces will usually be distributed in a complex, non-uniform manner, even when a homogeneous assembly of particles is subjected to uniform loading. This complex behavior is revealed in the photoelastic studies of two-dimensional disks reported by several investigators (Drescher, 1972; Oda and Konishi, 1974; Liu et al., 1995; Howell et al., 1999). The inhomogeneous distribution of optical fringe patterns that are observed in these studies, even for a uniformly applied load, shows that the load is transmitted by relatively rigid, heavily stressed chains of particles which

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form a sparse network of contacts, transmitting normal forces that are greater than the average. The remaining groups of particles, which separate the strong force chains, are only lightly loaded.

Although we are far from achieving a consensus on the nature of the distribution of contact forces in granular media or a perfect physical model to capture their micro-mechanics, some revealing results are found in recent numerical simulations on two-dimensional circular (Radjai et al., 1996, 1997, 1998; Radjai and Wolf, 1998) and three-dimensional spherical systems of particles under quasi-static loading (Radjai and Wolf, 1998; Thornton and Antony, 1998; Radjai et al., 1999; Antony, 2000). These studies demonstrate that the normal components of contact force provide the major contribution to the deviatoric stress and that the spatial distribution of contact forces can be divided into two sub-networks: (i) contacts carrying less than the average force (forming *weak force chains*), and (ii) contacts carrying greater than the average force (forming *strong force chains*). The contribution of the strong force chains to the deviator stress is dominant. These strong forces, carrying a greater than average normal contact force, are preferentially aligned in the major principal stress direction. Contacts that slide are predominantly within the weak force chains and they contribute primarily to the mean stress, with negligible contribution to the deviatoric stress. The weak force chains play a role similar to a supporting matrix surrounding a solid backbone of the strong force chains (Thornton and Antony, 1998; Radjai et al., 1999; Antony, 2000; Antony and Ghadiri, 2001). In the present investigation, we probe the effect of particle shape on the interplay among contact signatures, fabric, and bulk strength of three-dimensional particulate assemblies. We propose a simple fabric measure that correlates closely with bulk strength.

2. Methods

The current observations are based upon numerical simulations of triaxial compression that employ three assemblies of particles having similar densities, but with each assembly having a different particle shape: either spherical, oblate, or prolate (Fig. 1). The oblate and prolate shapes are solids of revolution, and more details of their numerical description can be found elsewhere (Wang et al., 1999; Kuhn, 2003). The three cubic assemblies each contained 4096 particles in initially dense packings (ovoids with coordination numbers 9.0–9.2 and solid fraction 0.73; spheres with coordination number 5.6 and solid fraction 0.66). Particle sizes in each assembly ranged from between 0.5 and 1.35 of the mean size (Fig. 2). Ratios of axial height to radial girth were randomly assigned to the oblate and prolate particles, within the ranges 0.65–0.85 (oblate) and 1.2–1.6 (prolate).

Slow, quasi-static triaxial compression simulations were carried out using the discrete element method (DEM) (Cundall and Strack, 1979). The advantage of using DEM to study particulate materials is its ability to give detailed information about the internal mechanics of large systems of particles, which we

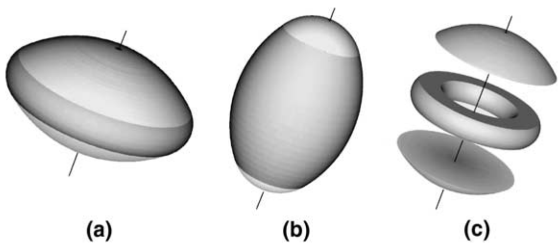

Fig. 1. Diagrams of an (a) oblate particle, (b) prolate particle, and (c) construction of an oblate particle from two spheres and a middle torus.

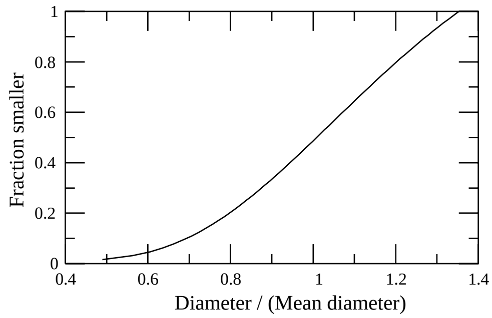

Fig. 2. Cumulative distribution of particle diameters, measured at the particle girth.

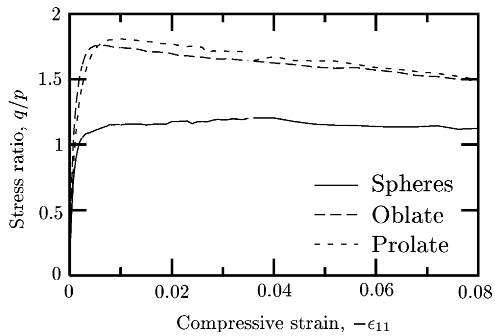

Fig. 3. Variation of deviator stress ratio q/p during triaxial compression for three particulate assemblies, each composed of different particle shapes.

explore in the current work. The method models the interactions between contiguous particles as a dynamic process, and the time evolution of the particles is advanced using an explicit central difference time integration scheme. A simple force mechanism was employed between contacting particles. Linear normal and tangential contact springs were assigned equal stiffnesses, and the coefficient of contact friction was 0.5.

The cubic assemblies were initially random, isotropic, and homogeneous, and the initial contact indentations were less than 0.02% of the particle size. During triaxial compression, the width of the assembly was slowly reduced at a constant rate in the 1–1 direction, while maintaining constant normal stresses, σ_{22} and σ_{33}, along the 2–2 and 3–3 sides. This loading arrangement permitted dilation of a dense assembly while it was being compressed in the single, 1–1 direction, which approximates the constant-stress loading conditions attained with the use of a rubber membrane in standard geotechnical triaxial tests, although we have used periodic boundaries. The loading was conducted slowly, so that the kinetic energy associated with velocity fluctuations was, on average, about 0.01% of the elastic energy in the contact springs. Fig. 3 shows the evolution of the three assemblies during the slow, progressive increase of compressive strain $-\epsilon_{11}$. The normalized strength q/p is a ratio of the deviator stress $q = \sigma_{11} - \sigma_{33}$ and the mean stress $p = \sigma_{kk}/3$. The strength curves for the oblate and prolate assemblies are quite similar, but both have higher values than that of the sphere assembly, a characteristic that has been documented in other studies (Rothenburg and Bathurst, 1992; Ouadfel and Rothenburg, 2001; Ng, 2001).

3. Analysis

Although one would expect that both the contact force distribution and the contact fabric influence the evolution of strength, this aspect of particulate materials is not yet well understood. As discussed earlier, the contacts that slide are predominantly among the weak force chains and they contribute primarily to the mean stress, whereas the strong force chains contribute to both the deviatoric and mean stress. Hence we intend to evaluate the contribution of those contacts having larger forces and the evolution of their fabric during deviatoric loading.

The average Cauchy stress $\bar{\sigma}_{ij}$ in a granular assembly can be directly computed as a sum of dyadic products associated with its M contacts:

$$\bar{\sigma}_{ij} = \frac{1}{V} \sum_{pq \in \mathscr{M}} l_i^{pq} f_j^{pq}, \tag{1}$$

where V is the assembly volume (Christoffersen et al., 1981). Each product is for a contact pq between particles p and q, and the pair pq is an element in the set $\mathscr{M}$ of all contacts. Branch vector $\mathbf{l}^{pq}$ connects a reference point on particle p to a reference point on particle q; and $\mathbf{f}^{pq}$ is the contact force exerted by q on p. Each vector can be expressed as the product of scalar magnitudes and unit directions, or

$$\bar{\sigma}_{ij} = \frac{1}{V} \sum_{pq \in \mathscr{M}} \ell^{pq} \left[f^{pq,\mathrm{n}} (m_i^{pq} n_j^{pq}) + f^{pq,\mathrm{t}} (m_i^{pq} t_j^{pq}) \right], \tag{2}$$

where $\mathbf{n}^{pq}$ is the outward unit normal of particle p at contact pq, $\mathbf{t}^{pq}$ is the unit tangent vector aligned with the tangential component of contact force $\mathbf{f}^{pq,\mathrm{t}}$, and $\mathbf{m}^{pq}$ and ℓ^{pq} are the direction and length of branch vector $\mathbf{l}^{pq}$:

$$\mathbf{f}^{pq} = f^{pq,\mathrm{n}} \mathbf{n}^{pq} + f^{pq,\mathrm{t}} \mathbf{t}^{pq} \tag{3a}$$

$$\mathbf{l}^{pq} = \ell^{pq} \mathbf{m}^{pq}. \tag{3b}$$

Thornton and his coworkers have measured the relative contributions of the normal and tangential contact forces, $f^{pq,\mathrm{n}}$ and $f^{pq,\mathrm{t}}$, toward the deviatoric stress $\bar{\sigma}_{ij} - \bar{\sigma}_{kk}/d$ in both two- and three-dimensional simulations ($d = 2$ or 3, as in Thornton and Barnes, 1986; Thornton and Antony, 1998). They found that the contribution of the normal forces is dominant, with the tangential forces producing only about 10% of the deviator stress. For the moment, we neglect the tangential forces and approximate the average stress $\bar{\sigma}_{ij}$ in (2) as

$$\bar{\sigma}_{ij} \approx \frac{1}{V} \sum_{pq \in \mathscr{M}} \ell^{pq} f^{pq,\mathrm{n}} \phi_{ij}^{pq} \tag{4}$$

with a contact fabric $\phi_{ij}^{pq} = m_i^{pq} n_j^{pq}$ associated with each pq contact.

On first sight, it might appear that $\bar{\sigma}_{ij}$ would closely correlate with the mean fabric – the average $\langle \phi_{ij}^{pq} \rangle$ of the M contacts. We found, however, that $\bar{\sigma}_{ij}$ and $\langle \phi_{ij}^{pq} \rangle$ are poorly correlated, whether correlation is measured among assemblies composed of different particle shapes, or it is measured for a single assembly at different stages of loading. This, perhaps, unfortunate result is due to cross-correlations among the ℓ^{pq}, $f^{pq,\mathrm{n}}$, and ϕ_{ij}^{pq} in (4). As has been mentioned, larger contact forces are preferentially aligned in the direction of the major principal (compressive) stress, which is closely aligned with the major eigenvector of fabric $\langle \phi_{ij}^{pq} \rangle$. Larger contact forces also tend to occur among larger particles, for which the lengths ℓ^{pq} are greater. (The coefficient of correlation between sets $f^{pq,\mathrm{n}}$ and ℓ^{pq} is, indeed, greater than that between sets $f^{pq,\mathrm{n}}$ and ϕ_{ij}^{pq}.)

A far better correlation between deviatoric stress and fabric is attained by admitting *only the most heavily loaded contacts*, as defined below, in the computation of an average fabric. Recalling that weak force chains contribute mainly to the mean stress, we rank and partition the M contacts into distinct subsets $\mathscr{M}_s$,

$s = 1, 2, \ldots, N_s$, such that $\mathscr{M} = \bigcup \mathscr{M}_s$. If the number N_s of subsets is sufficiently large, the average stress can be closely approximated as

$$\bar{\sigma}_{ij} \approx \frac{1}{V} \sum_{s=1}^{N_s} M_s \langle \ell^{\mathscr{M}_s} \rangle \langle f^{\mathscr{M}_s,\mathrm{n}} \rangle \langle \phi_{ij}^{\mathscr{M}_s} \rangle, \tag{5}$$

where averages are taken among the M_s contacts in each subset $\mathscr{M}_s$. Both (4) and (5) are approximations of the stress $\bar{\sigma}_{ij}$, since we have excluded the contribution of the tangential contact force components. With a selective partitioning of the contacts $\mathscr{M}$ into groups $\mathscr{M}_s$, the sets $\{\ell^{\mathscr{M}_s}\}$, $\{f^{\mathscr{M}_s,\mathrm{n}}\}$, and $\{\phi_{ij}^{\mathscr{M}_s}\}$ will be better correlated, and the estimate (5) will be better than that in (4). We now describe the partitioning.

A conventional approach has been to partition the M contacts according to their orientations (i.e., their fabrics ϕ_{ij}^{pq}, as in Oda, 1972; Bathurst and Rothenburg, 1990; Ouadfel and Rothenburg, 2001). We instead partition the contacts by ranking them according to the magnitudes of their normal forces $f^{pq,\mathrm{n}}$, and we assign a ranking parameter $s \in \{1/M, 2/M, 3/M, \ldots, 1\}$, or simply $s \in [0, 1]$, to each contact. For example, a contact pq with ranking $s = 0.15$ has a normal force $f^{pq,\mathrm{n}}$ greater than 15% of the contact population, but smaller than 85% of the population. The sum in (5) can then be written as

$$\bar{\sigma}_{ij} \approx \frac{M}{V} \int_0^1 \ell(s) f^n(s) \phi_{ij}(s)\, \mathrm{d}s, \tag{6}$$

noting that the tangential contact forces f^t have been neglected in this approximation. With this scheme, the cumulative stress contribution $\tilde{\sigma}_{ij}(s)$ of those contacts having a ranking $\acute{s}$ less than some value $s \in [0, 1]$ can be computed as

$$\tilde{\sigma}_{ij}(s) = \frac{M}{V} \int_0^s l_i(\acute{s}) f_j(\acute{s})\, \mathrm{d}\acute{s}. \tag{7}$$

4. Results

Our primary interest is in the deviatoric material response that was exhibited in the numerical simulations, and for this purpose we track a single measure of the deviatoric stress: the deviator stress $q = \bar{\sigma}_{11} - \bar{\sigma}_{33}$, also shown in Fig. 3. A corresponding deviator stress $\tilde{q}(s)$ can be computed with (7) and

$$\tilde{q}(s) = \tilde{\sigma}_{11}(s) - \tilde{\sigma}_{33}(s), \tag{8}$$

which is the cumulative contribution to the deviator stress q of those contacts having a ranking lower than s. Fig. 4 shows the cumulative contribution $\tilde{q}(s)$ of the ranked contacts to the full stress deviator q.

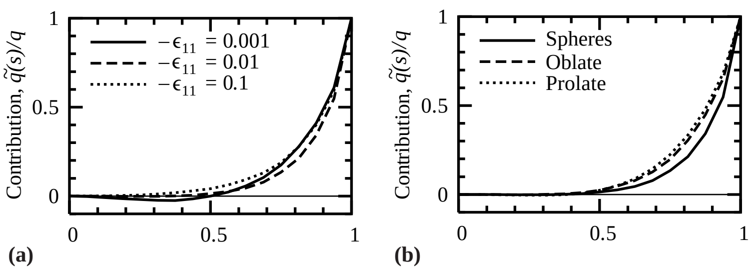

Fig. 4. Cumulative contributions of the s-ranked contact forces to the full deviator stress q: (a) spherical particles at three strains and (b) three particle shapes at strain $\epsilon = -0.01$.

Although difficult to distinguish (because some results overlay others), each plot is for three conditions. Fig. 4a shows the results for the triaxial compression of the sphere assembly at three strains ($-\epsilon_{11} = 0.001$, 0.01, and 0.1); whereas, Fig. 4b shows the results for three particle shapes at the same strain ($-\epsilon_{11} = 0.01$). The figure is further evidence that the 50% of most lightly loaded contacts (the weak chains, with $s < 0.5$) contribute little to the deviatoric stress. We also found that the distribution of contact fabrics $\phi_{ij}(s)$ among lightly loaded contacts ($s < 0.5$) can significantly change during triaxial loading. For this reason, the full, averaged fabric $\langle \phi_{ij}^{pq} \rangle$ correlates poorly with deviatoric stress. The results in Fig. 4 suggest, however, that

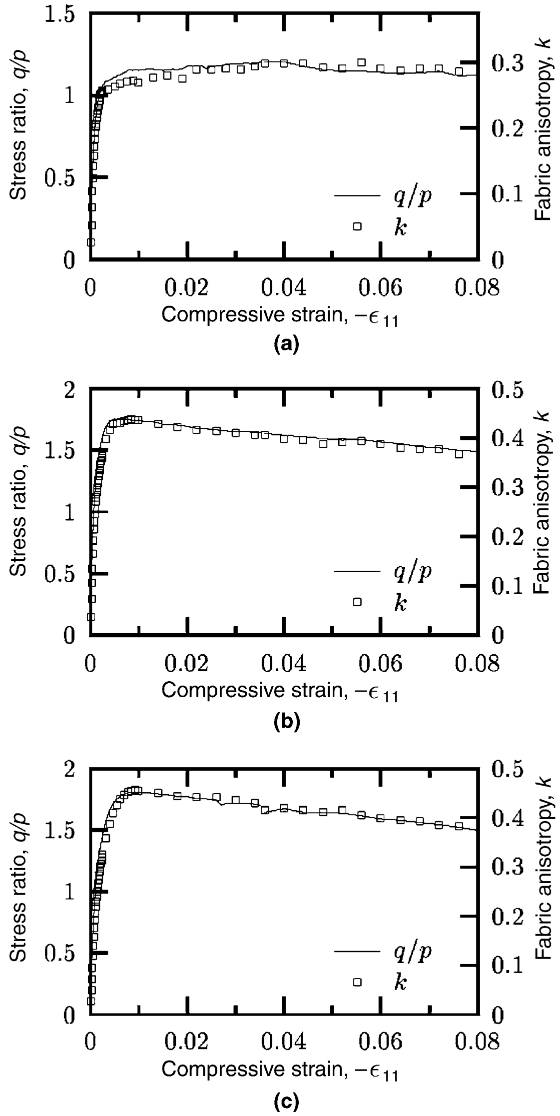

Fig. 5. Correlations among the fabric measure k and the deviator stress ratio q/p for three particle shapes. The same ratio $\alpha_k = k/(q/p) = 0.25$ has been used to scale the vertical axes in each plot: (a) spherical particles, (b) oblate particles and (c) prolate particles.

material fabric $\phi_{ij}(s)$ may be a good predictor of deviatoric stress, provided that we admit only the more heavily loaded contacts in our averaging of the ϕ_{ij}^{pq}.

In this regard, we use an alternative, restricted average fabric $\langle\phi_{ij}^{pq}\rangle_{s>0.5}$, suspecting that the fabric of those contacts with a normal force f^n greater than the median ($s > 0.5$) carries more information about the stress-bearing capacity of the material than the full average fabric $\langle\phi_{ij}^{pq}\rangle = \langle\phi_{ij}^{pq}\rangle_{s>0}$. The deviator of the restricted fabric measure is designated as k:

$$k = \langle\phi_{11}^{pq}\rangle_{s>0.5} - \langle\phi_{33}^{pq}\rangle_{s>0.5} = \frac{1}{1-0.5}\int_{0.5}^{1}[\phi_{11}(s) - \phi_{33}(s)]\,\mathrm{d}s, \tag{9}$$

where we take advantage of the axisymmetric conditions of the triaxial loading simulations, having $\langle\phi_{22}^{pq}\rangle_{s>0.5} = \langle\phi_{33}^{pq}\rangle_{s>0.5}$. This single measure of fabric is, indeed, a superior predictor of the deviator stress ratio q/p, as is shown in Fig. 5. Measured values of k are closely correlated with q/p for all three particle shapes and at all strains. The ratio $\alpha_k = k/(q/p)$ is, on average, about 0.25, and this ratio varies little with the particle shape or with strain: the standard deviation of α_k was only 0.016 across all conditions, although its variation was larger at small strains.

We have chosen the single criteria $s > 0.5$ for computing an effective fabric $\langle\phi_{ij}^{pq}\rangle_{s>0}$, but we have found that other, more restrictive criteria also lead to averaged fabrics that correlate well with the deviator stress. (e.g., the simple selection criteria $f^{pq,\mathrm{n}} > \langle f^{pq,\mathrm{n}}\rangle$ includes the 35–45% of most heavily loaded contacts and is also an excellent predictor of strength q/p). Our results show, however, that the fabric affects deviatoric stress only through the more heavily loaded contacts in a particulate material.

5. Conclusion

The results of this study provide some promise that a simple fabric measure can be used to characterize the evolution of stress in granular materials. Further studies are required to permit its more general use with other loading paths and under unloading conditions, with more elongated and non-smooth particle shapes, and with other contact characteristics.

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